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# HYDROCOOLING VEGETABLES

A Practical Guide  
to Predicting  
Final Temperatures  
and Cooling Times

UNITED STATES DEPARTMENT OF AGRICULTURE  
Agricultural Marketing Service  
Market Quality Research Division

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# HYDROCOOLING VEGETABLES

## A PRACTICAL GUIDE TO

### PREDICTING FINAL TEMPERATURES AND COOLING TIMES

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#### SUMMARY

Hydrocooling data in the form of half-cooling times or nomographs or both are presented for artichokes, asparagus, broccoli, Brussels sprouts, cabbage, cantaloups, carrots, cauliflower, celery, sweet corn, peas, potatoes, radishes, and tomatoes precooled in a flood-type hydrocooler under various conditions of exposure. These conditions were for the commodity completely exposed to the water, packed in containers, or jumble-stacked.

Instructions are given for using the half-cooling times and nomographs to predict the final commodity temperature when produce is hydrocooled under conditions of various commodity temperatures, water temperatures, and exposure times.

#### INTRODUCTION

Hydrocooling, the removal of field heat from fruits and vegetables by cold water, was used for vegetables in Florida in about 1923, and was introduced into California in about 1933 (6).<sup>2</sup> Although hydrocooling has been used for many years, much of the technical information available about the rate of cooling applies primarily to cooling that can be expected under only one or two specific sets of conditions of initial commodity temperature, water temperature, and duration of cooling. Information is limited on the amount of cooling that can be expected for specific commodities hydrocooled under various conditions of initial commodity temperature, water temperature, and exposure time.

The primary purpose of this paper is to provide information that will permit accurate predictions of final commodity temperatures when produce having various initial temperatures is hydrocooled in water of various temperatures for various lengths of time and under various exposures, and to present this information so that it will be easy to use. Data are presented for globe artichokes, asparagus, broccoli, Brussels sprouts, cabbage, cantaloups, carrots, cauliflower, celery, sweet corn, peas, potatoes, radishes, and tomatoes.

While the effect of hydrocooling on subsequent quality of the commodity was not a part of this study, other reports have shown hydrocooling to be an excellent method for precooling produce (4, 5, 8, 9, 10, 11).

Detailed information regarding the effect of hydrocooling on the quality of vegetables subject to chilling injury (for example, tomatoes) is limited; but it is doubtful if hydrocooling would cause chilling injury if the commodity were subsequently held at a temperature recommended for that particular commodity. Kasmire (personal correspondence) observed no quality differences between tomatoes submerged in 32° F. water for 1 hour and subsequently held 1 week at 68° F., and fruit held under the same conditions without prior hydrocooling. Srivastava et al. (7) found that tomatoes precooled to 32°-35° F. were in better condition after subsequent holding at 52°-55° F. than those held at this temperature without precooling.

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<sup>2</sup> Underlined numbers in parenthesis refer to items in Literature Cited, p. 7.



## MATERIALS AND METHODS

A shower or flood-type hydrocooler described previously (12) was used, and water temperatures were maintained at 32° to 35° F. Water flow rate was not varied in these tests, because previous work (12) had indicated that at high flow rates, comparable to those used in commercial hydrocoolers, the cooling rate was essentially independent of the flow rate. Therefore, a single, relatively high flow rate (16 gallons per square foot of cooler area per minute) was used in these experiments.

The flood-type hydrocooler was generally used in the conventional manner; but for peas (which are generally hydrocooled by submersion), submersion cooling also was tested. In the submersion tests, the container of produce was held under water in the reservoir tank of the hydrocooler used for the other tests, and water was circulated by operating the hydrocooler pump in the normal manner.

Each test was repeated three times, and the averages of the cooling constants are presented in the report. The data were analyzed statistically, using the analysis of variance and Duncan's Multiple Range Test (1).

Many of the more important vegetable crops of California and some of the less important ones were hydrocooled. All of the vegetables used were of a size commonly shipped from California.

Commodity temperatures were measured with a recording potentiometer with thermocouples (number 24 wire) which were 1/16 to 1/8 inch long. Where necessary, the thermocouples were tied in place during hydrocooling, and vaseline was used to seal the opening in the tissue made by the thermocouple. This procedure prevented the entrance of water and the recording of erroneous temperatures. Water temperatures were taken either with a potentiometer and thermocouples or a mercury thermometer.

Temperatures were measured and evaluated so as to be representative of the temperature of the entire commodity. For round, oval, or cylindrically shaped vegetables, temperatures were usually measured halfway to the center of the commodity, as previous work with cantaloups (12) had shown this to be representative of the commodity as a whole. In some commodities, thermocouples were inserted to the center rather than halfway to the center of a tissue, either because of its small size or because of the difficulty in locating any other point that would provide a representative temperature. For cauliflower, broccoli, artichokes, celery, and radishes, with more than one type of tissue (such as the curd, stem, and leaves of cauliflower), temperatures in the various tissues were observed separately. These temperatures were then averaged according to the proportion of the whole represented by the particular tissue. The proportions were determined by trimming and weighing the various tissues from at least 10 samples of each commodity. These proportions are referred to as "weighting factors" in table 1. Rates of cooling representative of the commodity as a whole were thus obtained.

Cooling of the commodities was studied when they were packed in containers and when they were jumble-stacked as they might be on a conveyor belt or in a jumble-pack.

In such vegetables, temperatures were taken in the bottom and top layers and also in one or two intermediate layers. The cooling of each commodity was also determined in a single layer (or in the top layer of a container with the lid off) where the commodity was completely exposed to the water. This represents the maximum exposure and, therefore, the fastest possible cooling.



## PRESENTATION AND APPLICATION OF DATA

**Nomographs.** -- The cooling data are presented in the form of nomographs for general on-the-job use and in the form of mathematical constants for detailed comparisons of rates of cooling under various specified conditions. For most vegetables, nomographs were constructed for the commodity completely exposed to the water and for the commodity packed in a container or jumble-stacked (figs. 1 through 18). A single nomograph is sometimes used to represent the cooling of a commodity under different conditions, when the cooling curves under these conditions are essentially the same. A nomograph for cabbage is not presented because the half-cooling time is so great that hydrocooling is impractical for cabbage.

The time scale for the nomographs was constructed by determining the percent of temperature reduction at various times during hydrocooling, and converting a linear scale, showing percent of cooling, directly to a time scale. The time scale was then placed on the lower horizontal axis of a graph, the scale of percent of cooling on the upper horizontal axis, and the temperature in degrees F. on the vertical axis. The scale showing percent of cooling was retained on the nomographs to show the proportion of heat removed at various times during hydrocooling. The time and temperature scales are used to predict final commodity temperatures if the initial temperature, the water temperature, and the time in the hydrocooler are known. Conversely, the cooling times may be predicted if the desired final temperature, the initial temperature, and water temperature are known. The time scales are specific for each commodity and each condition.

To use one of the nomographs, a straightedge is placed from the initial commodity temperature at zero time to the water temperature at infinite time and the final temperature read at any cooling time along the straightedge; or, conversely, the cooling time required to reach any desired final temperature also may be read along the straightedge. Following are examples of how cooling predictions can be made with the nomographs.

**PROBLEM:** Asparagus at  $90^{\circ}$  F. is packed in crates and hydrocooled. (a) How long must it be cooled with  $33^{\circ}$  water to reach  $40^{\circ}$ ? (b) If the water temperature is  $37^{\circ}$ , how long will it take to reach  $40^{\circ}$ ? (c) What will the temperature be after 4 minutes of cooling in  $37^{\circ}$  water?

**SOLUTION:** Locate the nomograph for crated asparagus (fig. 3). (a) Place a straightedge from  $90^{\circ}$  on the left to  $33^{\circ}$  on the right. Read the time at  $40^{\circ}$ . Answer: 7 minutes. (b) Place a straightedge from  $90^{\circ}$  to  $37^{\circ}$ . Read the time at  $40^{\circ}$ . Answer: 12 minutes. (c) Leave straightedge from  $90^{\circ}$  to  $37^{\circ}$ , read the temperature at 4 minutes. Answer:  $51^{\circ}$  F.

The half-cooling time is used to characterize the cooling process because, theoretically, it is independent of the initial temperature and remains constant throughout the cooling period. As the name indicates, it is the time required to reduce the temperature difference (commodity temperature minus water temperature) by one-half. When the half-cooling time has been determined, the cooling that can be accomplished in various times and with various commodity and water temperatures can be predicted. A detailed discussion of cooling constants is given in the appendix.

Half-cooling times were determined for each vegetable under various conditions, and these are given in tables 2 through 14. To simplify the use of half-cooling times, a general nomograph (fig. 19) similar to the specific nomographs described above was constructed. This graph is used in the same manner as the specific nomographs except that the time scale is given in number of half-cooling periods rather than in minutes. To convert from half-cooling periods to time, the half-cooling time for a particular commodity is multiplied by the number of half-cooling periods read from the graph.

For example, assume that topped radishes with a half-cooling time of 2.2 minutes (table 13) are to be hydrocooled using  $32^{\circ}$  F. water.



PROBLEM: How long would it take to cool the radishes from 80° F. to 50° F. ?

SOLUTION: Place a straightedge on the general nomograph (fig. 19) as explained previously. We find that it would take 1.4 half-cooling periods. Multiply the number of half-cooling periods (1.4) by the half-cooling time (2.2 minutes). Answer: 3.1 minutes.

To convert from time to half-cooling periods, divide the time by the half-cooling time.

Cooling of the surface of a commodity begins immediately when the commodity is in contact with cold water, but cooling within a commodity may not begin until later. This delay between the time hydrocooling begins and the time at which the temperature within the commodity begins to drop is called the lag time.

When using half-cooling times in mathematical calculations, or with the general nomograph, lag times, when given, should be used for greatest accuracy. When using the general nomograph to predict the time that will be required to cool a commodity with a lag time to a specific temperature, proceed as before; but when the time has been determined from the graph, add the lag time. If the commodity temperature after a given time is desired, subtract the lag time from the time, then use the remainder with the general nomograph as described above.

For some commodities, the average temperature drops very rapidly at the beginning of the cooling period (because of relatively small, fast-cooling tissues), then more slowly as the bulky, slower cooling portions are cooled. Two half-cooling constants are required to describe such data accurately, but a combined half-cooling time given in the tables (tables 2, 4, 7, 8, 9, 11, and 13) and used with the general nomograph (fig. 19) may be adequate for most purposes. For those interested in greatest accuracy, the two half-cooling constants also given in the tables and discussed in the appendix should be used where specific nomographs are not presented.

## DISCUSSION

Since packaging affects the flow of water around a commodity, it may influence the rate of cooling. Cooling was usually slower for a commodity hydrocooled in a lidded crate than for one completely exposed, because the flow of water into the package was restricted (asparagus, celery, sweet corn). Cooling was usually more rapid in the top layer than in the lower layers. However, if the flow of water out of the package was restricted, and it filled with water, the cooling was much more rapid in the lower layers and was usually not greatly different from the cooling in the top layer (broccoli, Brussels sprouts, radishes). For optimum circulation and cooling in the lower layer, most of the water should drain from the bottom rather than over the top of the container (2).

The nomograph for a single layer of sweet corn (fig. 12) may be used for cantaloups, because previously published data (12) show that the cooling curve and the half-cooling time of cantaloups were identical to those of sweet corn.

If, in commercial practice, adequate time and temperature data indicate cooling appreciably different from that predicted from the nomographs, the reasons should be determined. The most likely sources of disagreement between temperatures predicted from the nomographs and those observed under commercial conditions are an inadequate or poorly distributed flow of water, a commodity exposure different from that used in the test, a different average commodity size, or temperatures measured at different points from those used in the tests.

With due consideration for all variables between experimental and commercial hydrocooling, the average temperatures predicted from the nomographs are considered to be a reliable guide to commercial hydrocooling if the initial commodity temperature, water temperature, and cooling time are accurately known, and if the appropriate

(commodity and exposure) nomograph or half-cooling time is used. However, if speed of cooling in a commercial hydrocooler is proved to be adequate, but different from the cooling predicted from the nomographs, sufficient accurate data should be obtained to determine the percent of cooling at different times, and a new nomograph constructed for that particular commercial operation.

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## APPENDIX

## Nomographs and Tables

TABLE 1.--Location of thermocouples in vegetable tissues during hydrocooling and the weighting factors of the tissues

Commodity	Tissue	Diameter	Thermocouple location		Weighting factor <sup>1</sup>
			Depth	Approximate position	
		<u>Inches</u>	<u>Inches</u>		<u>Percent</u>
Artichokes	Receptacle		3/4 (at 45° angle up from base of bud)	Center of receptacle	46
	Bracts		1/8	Near base of 4th or 5th bract	54
Asparagus	Butt	5/8	1/4	Center of spear	33
	Middle	1/2	1/4	Center of spear	33
	Tip	3/8	1/8	Center of spear	33
Broccoli	Main stalk	1-1/4	5/16	Halfway to center of stalk	46
	Branch and curd (branch)	3/8	3/16	Center of branch	54
Cauliflower	Stem		1-1/2	Center of stem	15
	Curd and branches		1/2	Center of curd	66
	Leaf		--	Center of midrib	19
Celery	Butt	3	1-1/2	Center of butt	27
	Petiole and leaf		--	Center of intermediate-size petiole	73
Radishes	Root	1	1/4	Halfway to center of root	60
	Leaf		--	Random position among leaves	40

<sup>1</sup> Percent of commodity (by weight) represented by given tissues.



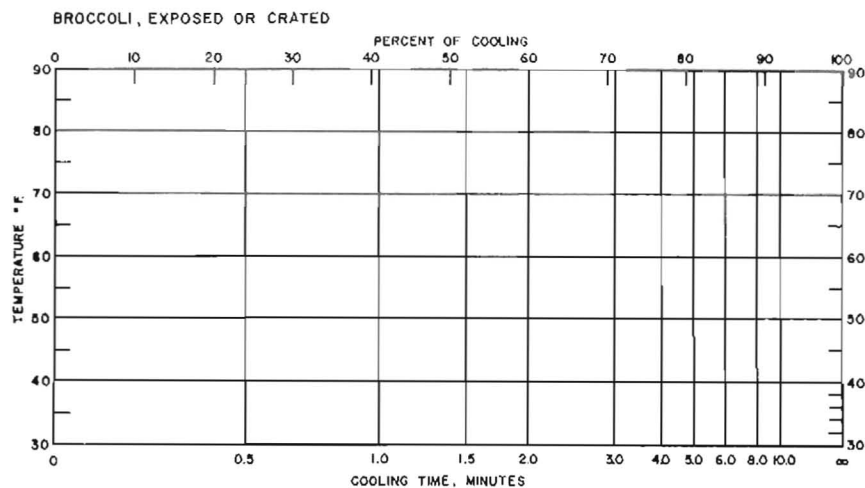


Figure 4.--Hydrocooling nomograph for broccoli, either completely exposed or in the pony broccoli crate with the lid off and the paper liners open at the top.

TABLE 4.--Cooling constants for broccoli

Commodity exposure during hydrocooling		Cooling constants		
Container	Layer	Proportion (P)	Half cooling time (Z)	Combined half-cooling time (Z <sub>c</sub> )
			<u>Minutes</u>	<u>Minutes</u>
Pony broccoli crate, lid off, 2 paper liners, open at top (crate filled 3/4 with water)	1st (top)	0.43	4.5	<sup>1</sup> 2.4 ab
		0.57	0.8	
	2nd	0.30	4.2	1.7 a
		0.70	0.7	
	3rd	0.40	4.2	2.2 ab
		0.60	0.8	
	4th (bottom)	0.47	4.4	2.5 ab
		0.53	0.9	
Pony broccoli crate, lid off, no paper liners	1st (top) (completely exposed)	0.30	5.0	2.1 ab
		0.70	0.9	
	2nd	0.38	6.0	3.0 ab
		0.62	1.2	
	3rd	0.50	5.8	3.4 bc
		0.50	1.1	
	4th (bottom)	0.51	6.2	3.9 c
		0.49	1.4	
Completely exposed		0.30	5.0	2.1
		0.70	0.9	
Crate with liner	average	0.40	4.3	2.2 a
		0.60	0.8	
Crate without liner	average	0.42	5.8	3.1 b
		0.58	1.2	

<sup>1</sup> Means within a box not followed by the same letter are significantly different at the 1 percent level.

TABLE 5.--Cooling constants for Brussels sprouts<sup>1</sup>

Commodity exposure during hydrocooling		Cooling constants	
Container	Layer	Proportion (P)	Half-cooling time (Z)
			<u>Minutes</u>
Carton, lid open (9" deep) (Carton filled with water)	Top	1.0	<sup>2</sup> 4.6 a
	Middle	1.0	4.5 a
	Bottom	1.0	5.2 a
Jumble stack (9" deep)	Top (completely exposed)	1.0	4.4 a
	Middle	1.0	6.8 b
	Bottom	1.0	6.8 b
Completely exposed		1.0	4.4
Carton average		1.0	4.8 a
Jumble-stack average		1.0	6.0 b

<sup>1</sup> Thermocouple 1/3 inch deep, approximately halfway to the center of the 1-1/3-inch-diameter sprouts.

<sup>2</sup> Means within a box not followed by the same letter are significantly different at the 1 percent level.

TABLE 6.--Cooling constants for cabbage<sup>1</sup>

Commodity exposure during hydrocooling	Cooling constants	
	Lag Time	Half-cooling time (Z)
	<u>Minutes</u>	<u>Minutes</u>
Jumble stack, top layer (completely exposed)	10	69
Carton, lid open (carton filled with water) (average, 2 layers)	<sup>2</sup> --	81
Jumble stack (average, 4 layers)	13	81

<sup>1</sup> Thermocouple 1-1/2 inches deep, approximately halfway to the center of the 6-inch-diameter heads. It is regarded as impracticable to hydrocool cabbage, so no nomograph for it is presented.

<sup>2</sup> Because water filled carton and entered hole containing thermocouple, temperatures only taken initially and after cooling (60 minutes). Lag time could not be determined.



TABLE 8.--Cooling constants for trimmed cauliflower

Commodity exposure during hydrocooling	Cooling constants		
	Proportion (P)	Half-cooling time (Z)	Combined half-cooling time (Z <sub>c</sub> )
Completely exposed in single layer		<u>Minutes</u>	<u>Minutes</u>
	0.5	12.5	7.2
	0.5	1.8	

TABLE 10.--Cooling constants for sweet corn in husks (5-dozen size)<sup>1</sup>

Commodity exposure during hydrocooling		Cooling constants	
Container	Layer	Proportion (P)	Half-cooling time (Z)
			<u>Minutes</u>
None (completely exposed)	Single	1.0	<sup>2</sup> 20 a
Wirebound corn crate, lidded	1st (top)	1.0	25 a
	3rd (middle)	1.0	35 b
	5th (bottom)	1.0	25 a
Completely exposed		1.0	20 a
Crate average		1.0	28 b

<sup>1</sup> Thermocouple 9/16 inch deep, approximately halfway to the center of the 2-1/8-inch-diameter ears (at midlength).

<sup>2</sup> Means within a box not followed by the same letter are significantly different at the 1 percent level.



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# GENERAL NOMOGRAPH

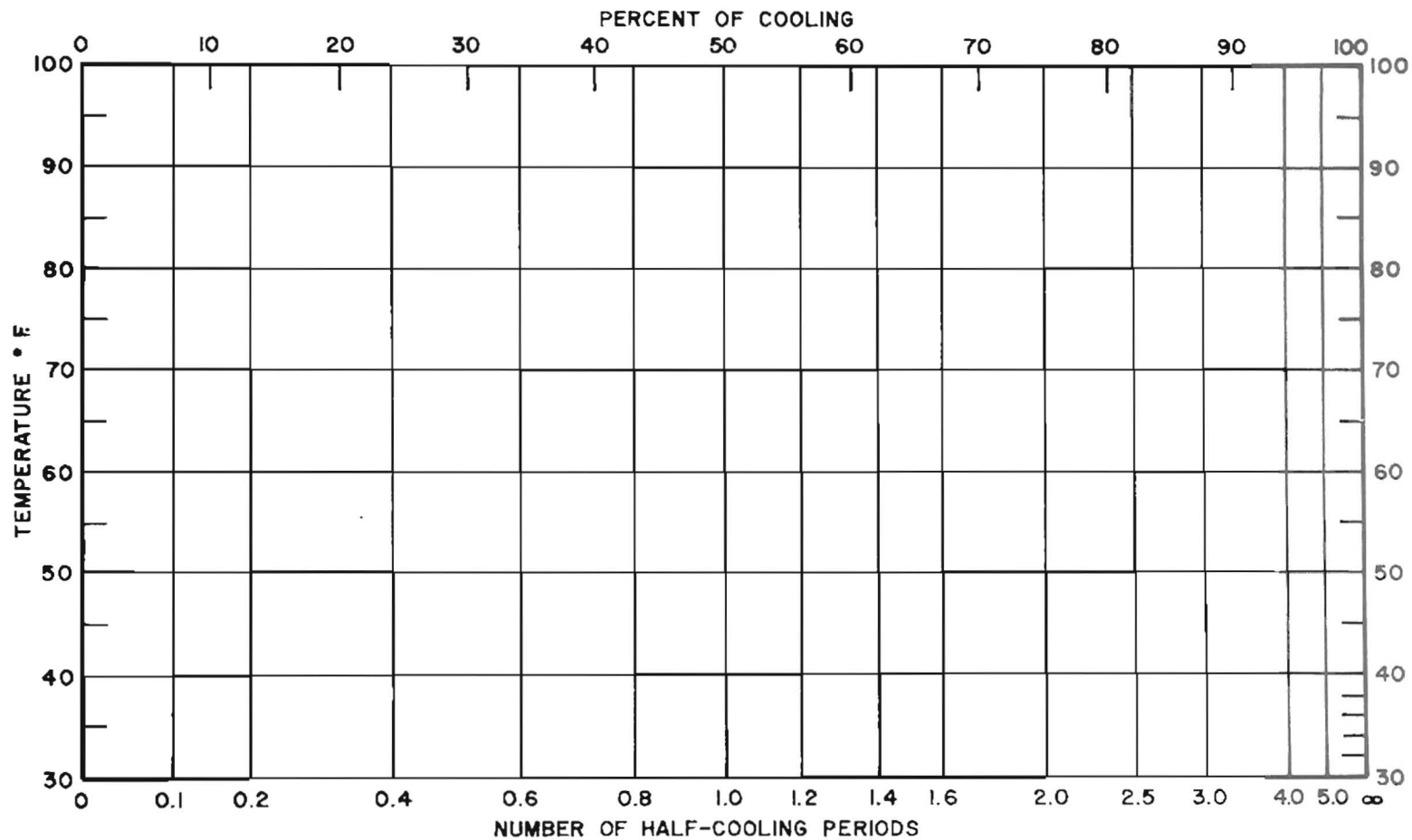


Figure 19. --General nomograph to be used with half-cooling times.

## Equations

The rate of cooling is dependent upon certain characteristics of a vegetable, such as its density, heat capacity, and heat conductivity, which are usually constant for a given commodity. Cooling is also dependent on the difference in temperature between the object and its surroundings. As the object cools, the temperature difference changes and, therefore, the rate of cooling changes continuously with time. The following equation<sup>3</sup> describes the cooling process (2):

$$\frac{T - T_w}{T_i - T_w} = e^{-kt} \quad \text{Equation 1}$$

Under certain conditions, deviations from this mathematical model may occur, usually because of the size and shape of the object or variable temperature of the cooling water. Deviations occur when temperatures are observed near the surface of the object undergoing cooling. A rapid initial drop in temperature, characteristic of the surface, is followed by a slower cooling rate characteristic of the remainder of the object. This process may be described by the compound expression: (3, 13)

$$\frac{T - T_w}{T_i - T_w} = P_1 e^{-k_1 t} + P_2 e^{-k_2 t} \quad \text{Equation 2}$$

where  $P_1$  and  $P_2$  are the apparent proportions of the object that is characterized by the corresponding cooling coefficient. These constants are derived from the cooling data and have nothing to do with the "weighting factors" described previously. The constants are simply mathematical devices which permit accurate and economical description of the data.

If the temperature is observed near the center of the object, a time lag occurs before the temperature at this point begins to fall. This process may be expressed as follows:

$$\frac{T - T_w}{T_i - T_w} = e^{-k(t-a)} \quad \text{Equation 3}$$

where  $a$  is the lag time. This expression may be written in the same form as equation 2:

$$\frac{T - T_w}{T_i - T_w} = P e^{-kt}, \text{ where } P = e^{ak} \quad \text{Equation 3a}$$

Several workers have recommended the use of the half-cooling time  $Z$  rather than the cooling coefficient  $k$  for characterizing the cooling process (2, 3, 13).

$$\frac{T - T_w}{T_i - T_w} = P_1 e^{-0.693t/Z_1} + P_2 e^{-0.693t/Z_2} \quad \text{Equation 4}$$

$$k = 0.693/Z$$

<sup>3</sup> Symbols used in this section are:  $T$ , the commodity temperature;  $T_i$ , the initial commodity temperature;  $T_w$ , the water temperature;  $T_a$ , the apparent initial temperature;  $P$ , the ratio of apparent to actual initial temperature;  $t$ , the cooling time;  $k$ , the cooling coefficient;  $a$ , the lag time;  $Z$ , the half-cooling time;  $n$ , the number of half-cooling periods; and  $e$ , the base of natural logarithms.



The values of the constants  $P_1$ ,  $P_2$ ,  $Z_1$ , and  $Z_2$  were determined by graphical methods. The temperature difference between the commodity and the cooling water ( $T - T_w$ ) was plotted versus the exposure time ( $t$ ) on semi-logarithmic paper. The value of  $Z_1$  was determined from the lower straight-line portion of the curve by observing the time required to reduce the temperature difference by one half. The straight-line portion of the curve was then extrapolated back to zero time to find the apparent initial temperature ( $T_a$ ). The value of  $P_1$  is  $T_a / (T_i - T_w)$  and the value of  $P_2$  is  $1 - P_1$ . The extrapolated curve was then subtracted from the observed curve and a second set of points plotted which usually approximated a straight line. The value of  $Z_2$  was determined from a straight line through these points.

Since it seemed awkward to compare sets of two half-cooling times, single values were calculated by adding the values of  $Z_1$  and  $Z_2$  multiplied by  $P_1$  and  $P_2$  respectively.

$$Z_c = P_1 Z_1 + P_2 Z_2$$

To use the constants given in the tables the initial temperature difference (commodity temperature minus the coolant temperature) and  $1/2$  and  $1/4$  of the initial temperature difference are plotted on the vertical log scale of semi-logarithmic paper versus 0, 1, and 2 half-cooling times, respectively, in minutes on the horizontal scale. The temperature difference after any cooling time or the time required to reach any temperature difference may then be determined from the graph. If a commodity is characterized by two half-cooling times, the initial temperature difference is multiplied by the proportion corresponding to each of the half-cooling times, plotted as explained above, and then the individual lines are summed to give the desired curve.

Nomograph time scales may be constructed by converting a percentage of cooling scale directly to time from experimental data as described in a previous section. An alternative method may be derived from equation 1, and it illustrates the relation between this scale and the more conventional logarithmic scale.

$$\frac{T - T_w}{T_i - T_w} = e^{-0.693t/Z} \quad \text{Equation 5a}$$

$$= \frac{1}{2^{t/Z}} \quad \text{Equation 5b}$$

$$= \frac{1}{2^n} \quad \text{Equation 5c}$$

Equation 5c may be rearranged and expanded as follows:

$$T - T_w = \frac{T_i - T_w}{2^1} + \frac{T_i - T_w}{2^2} + \dots + \frac{T_i - T_w}{2^n} = T_i - T_w$$

$$n = 1, 2, 3, \dots,$$

If the total cooling is taken as 100 percent, the expression becomes:

$$T - T_w = \frac{100}{2} + \frac{100}{4} + \dots + \frac{100}{2^n} = 100$$

The succeeding terms in the expansion may be evaluated and marked off on a scale from 0 to 100 and the successive points identified in terms of half-cooling periods. The  $\underline{n}$  scale in figure 19 is an example which is perfectly general for cooling, simple diffusion, and first-order chemical reactions. This scale may be converted to a specific time scale by multiplying the values of  $\underline{n}$  by the half-cooling time for a specific commodity and substituting the resulting times for  $\underline{n}$  on the scale. In this form, the scale is specific for a particular half-cooling time.